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Chiral (γ_5) Symmetry of Massless Fields and Neutrino Theory of Photons

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Abstract

It is shown that the field theory of massless particles has the internal symmetry group $G = U(1) \otimes \mathrm{Aut}\, U(1)$. The γ_5 and dual transformations of neutrino and electromagnetic fields are particular cases of the transformations of this group. The *CP* transformations of massless field can also be included in the transformations of this group. The following formulation of Pryce (1938) theorem is given: Statistical properties of the composite photon in the neutrino theory of light are inconsistent with chiral (γ_5) symmetry of neutrino and electromagnetic fields.

1. *Introduction*

As has been mentioned in the literature (see, for example, Takabayasi, 1959; Wheeler, 1960; Pestov, 1974), the γ_5 transformations of neutrino field

$$\psi \to \exp(i\theta\gamma_5)\psi, \qquad \bar{\psi} \to \exp(-i\theta\gamma_5)\bar{\psi} \tag{1.1}$$

and dual transformations of electromagnetic field

$$\mathbf{F} \to \exp(i\theta)\mathbf{F}, \qquad \mathbf{F}^* \to \exp(-i\theta)\mathbf{F}^* \tag{1.2}$$

where $\mathbf{F} = \mathbf{E} + i\mathbf{H}$, $\mathbf{F}^* = \mathbf{E} - i\mathbf{H}$, are rather alike.

In particular, Wheeler (1960) has formulated a question about the reason for this analogy. The problem can be solved if the theory of massless particles has some internal symmetry group, the particular cases of the transformations of which will be (1) and (2) transformations. Of course, the physical reason for the existence of such a group must also be clarified. We shall show that this group exists and that its appearance is inseparably linked with the intrinsic space-time properties of massless particles: axial symmetry about an axis that coincides with the direction of momentum of the particle.

On the basis of the above-mentioned conclusions the analysis of the neutrino theory of light (or photons) is given. The neutrino theory of light was first suggested by de Broglie and was developed further by Jordan and Kronig.[1] Pryce (1938) pointed out (from the analysis of the properties of the solutions of the Dirac equation with $m = 0$) that the state of a photon in the neutrino theory of light is not invariant under rotation, about the direction of neutrino momentum, of a certain vector $\mathbf{n}$ that appears in the theory in constructions of this state. But the choice of the direction of this vector $\mathbf{n}$ must be arbitrary, and this fact speaks in favor of inconsistency of the theory. It will be shown in our paper that this result (the Pryce theorem) can be formulated as a condition of compatibility of the supposed statistical properties of massless particles and internal symmetry of the theory of massless fields.

2. *Symmetry of Massless Fields*

We shall define the transformation properties of the operator ψ of massless fields under a group $U(1)$:

$$U_\theta \psi(x) U_\theta^{-1} = \exp(i\theta\Pi)\psi(x)\exp(-i\theta\Pi) = \exp\{2i\theta(j_1 - j_2)\}\psi(x) \quad (2.1)$$

where $\psi(x)$ is transformed in accordance with the irreducible representation (IR) (j_1, j_2) of the Lorentz group. The choice of the possible IR of ψ is restricted by the following condition:

$$j_1 - j_2 = \lambda \quad (2.2)$$

where λ is a helicity of a massless particle.

We have from (2.1) that

$$[\Pi, \psi(x)] = 2(j_1 - j_2)\psi(x) \quad (2.3)$$

where Π is a generator of transformations of the group $U(1)$.

The IR (j_1, j_2) of the lowest dimensions that satisfy the condition (2.2) are the $(0, j)$ and $(j, 0)$. The fields $\varphi(x)$ and $\chi(x)$, which are transformed according to IR $(0, j)$ and $(j, 0)$, describe the massless particles with helicity $\pm\frac{1}{2}, \pm 1, \ldots$ under $j = \frac{1}{2}, 1, \ldots$.

There exist some approaches to the formulation of the field theory of massless particles (see, for example, Dowker, 1973, and literature cited there). For our purpose the most convenient is considered to be the formulation that was given by Weinberg (1964a, b; 1965). (Naturally, the results achieved in our paper take place in other formulations.)

[1] In papers by Pryce (1938) and Berezynsky (1966) one finds the literature on this subject.

In this case the fields $\varphi(x)$, $\chi(x)$ satisfy the following field equations:

$$\begin{aligned}\left(\mathbf{J}^j \nabla - j\frac{\partial}{\partial t}\right)\varphi(x) = 0 \\ \left(\mathbf{J}^j \nabla + j\frac{\partial}{\partial t}\right)\chi(x) = 0\end{aligned} \tag{2.4}$$

and canonical commutation relations

$$[\varphi_\alpha(x), \varphi^+_{\alpha'}(x')]_\pm = F_{\alpha\alpha'}(x, x'),\ [\chi_\alpha(x), \chi^+_{\alpha'}(x')]_\pm = \bar{F}_{\alpha\alpha'}(x, x') \tag{2.5}$$

where J^j is the matrix in the corresponding representations, $F_{\alpha\alpha'}$, $\bar{F}_{\alpha\alpha'}$ are some singular functions, the manifest expressions of which can be found in papers by Weinberg (1964b). Signs ($\pm$) in (2.5) mean commutators of anti-commutators that depend on statistics which the particles obey and which are described by the fields $\varphi(x)$, $\chi(x)$. These are only nonvanishing commutators (or anticommutators) between $\varphi(x)$, $\varphi^+(x)$, $\chi(x)$, $\chi^+(x)$.

The transformations (2.1) commutate with the transformations of the Lorentz group, they are not connected with space-time transformations and are canonical transformations, leaving the commutation relations and equations of motion to be invariant.

Combining the fields $\varphi(x)$ and $\chi(x)$ [i.e., making the transition to the $2(2j + 1)$ formulation], we can put down the transformations (2.1) for the representation $(0, j) \oplus (j, 0)$ in the following form:

$$U_\theta \Psi(x) U_\theta^{-1} = \exp(2ij\gamma_5\theta)\Psi(x) \tag{2.6}$$

where $\gamma_5 = \binom{0\ I}{-I\ 0}$, $\Psi = \binom{\varphi}{\chi}$, and I is a single matrix in the corresponding representation. The transformations (2.6) under $j = \frac{1}{2}, 1$ correspond to γ_5 and dual transformations of neutrino and electromagnetic fields. At the same time equations (2.4) and the commutation relations (2.5) describe accordingly the Weyl equation and the Maxwell equations and the corresponding commutation rules. So, the field theory of massless particles has an internal (chiral) symmetry group $U(1)$ the particular cases of which are γ_5 and dual symmetries of neutrino and electromagnetic fields.

Dual symmetry of electromagnetic field is not known as well as γ_5 symmetry of neutrino field and it is reasonable to give here a brief discussion of this subject. In contrast to the Lagrangian density of neutrino field the Lagrangian density of electromagnetic field is not invariant under the dual transformations

$$F_{\mu\nu}^2 \to F_{\mu\nu}^2 \cos 2\theta + F_{\mu\nu}\tilde{F}_{\mu\nu} \sin 2\theta \tag{2.7}$$

where

$$\tilde{F}_{\mu\nu} = -(i/2)\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta},\ \tilde{\tilde{F}}_{\mu\nu} = -F_{\mu\nu},\ \epsilon_{1234} = 1$$

$\epsilon_{\mu\nu\alpha\beta}$ is the completely antisymmetric Ricci tensor. But taking into consideration the expression of the field tensor $F_{\mu\nu}$ through the electromagnetic potentials A_μ

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.8}$$

it can be seen that the expression $F_{\mu\nu}\tilde{F}_{\mu\nu}$ can be cast in the form of divergence of some quantity (see, for example, Landau and Lifschitz, 1968). The factor $\cos 2\theta$ of $F_{\mu\nu}^2$ does not depend on the coordinates or the variables of the system. In accordance with Noether's theorem (see, for example, Hill, 1951) such transformations of the Lagrangian can be accepted. In this case the law of conservation of some quantity can be found. In the case under consideration the conserved quantity (dual "current") has the following form (Strazhev, 1968, 1970; see also Strazhev and Tomilchik, 1973):

$$\begin{aligned} \Pi_\mu &= F_{\mu\nu}B_\nu - \tilde{F}_{\mu\nu}A_\nu \\ \tilde{F}_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{2.9}$$

The unitary operator of dual transformations (2) has the form (Zwanziger, 1968; Strazhev, 1968)

$$U(\theta) = \exp\left\{ i\frac{\theta}{2} \int [\mathbf{A}(x)\mathbf{H}(x) - \mathbf{B}(x)\mathbf{E}(x)]d^3x \right\} \tag{2.10}$$

where $\mathbf{H} = \nabla \times \mathbf{A}$, $\mathbf{E} = -\nabla \times \mathbf{B}$, and $\mathbf{A}$, $\mathbf{B}$ are transversal potentials that describe the radiation field. The generator of dual transformations Π corresponds to the fourth component of the Π_μ and is proportional to the difference between the number of the right and left circularly polarized photons. The dual transformations can be rewritten in the following form:

$$\begin{aligned} U_\theta \Phi_i U_\theta^{-1} &= \Phi_i \cos\theta + \tilde{\Phi}_i \sin\theta \\ U_\theta \tilde{\Phi}_i U_\theta^{-1} &= -\Phi_i \sin\theta + \tilde{\Phi}_i \cos\theta \end{aligned} \tag{2.11}$$

where $i = 1, 2$; $\Phi_1 = \mathbf{E}$, $\tilde{\Phi}_1 = \mathbf{H}$, $\Phi_2 = \mathbf{A}$, $\tilde{\Phi}_2 = \mathbf{B}$. As a particular case of transformations (2.11) under $\theta = \pi/2$ we have the well-known transformations of Larmor: $\mathbf{E} \to \mathbf{H}$, $\mathbf{H} \to -\mathbf{E}$.

3. *Physical Meaning of $U(1)$ Symmetry*

In spite of the formal resemblance between the transformations (2.1) and the usual phase transformations of the field operators, there exists an important difference between them: The generators of the phase transformations (the gauge transformations of the second kind) in the quantum theory are connected with the generators of the superselection rules. Their action on the field quantities does not depend on the kind of IR of the Lorentz group in accordance with which they are transformed. But the last property is not the property of the generator of chiral (γ_5) transformations, as one can see from (2.3).

Besides, we must use the Hermitian operators for the description of the purely neutral particles that are not to be the subject of the phase but of the chiral transformations. If we express the operators $\varphi(x)$, $\chi(x)$ for the case of purely neutral fields through the annihilation and creation operators $A^+(\lambda, k), A(\lambda, k)$ (see Weinberg, 1964, 1965), one can see that the following transformation properties of the operators A, A^+ are in accordance with (2.1):

$$\begin{aligned} U_\theta A(\lambda, k) U_\theta^{-1} &= \exp(-2i\lambda\theta) A(\lambda, k) \\ U_\theta A^+(\lambda, k) U_\theta^{-1} &= \exp(2i\lambda\theta) A^+(\lambda, k) \end{aligned} \tag{3.1}$$

The helicity states $|\lambda, p\rangle$ have the following transformation rule:

$$\begin{aligned} U_\theta |\lambda, p\rangle &= \exp(2i\lambda\theta) |\lambda p\rangle \\ \Pi |\lambda p\rangle &= 2\lambda |\lambda, p\rangle \end{aligned} \tag{3.2}$$

It is seen from (3.1) and (3.2) that the chiral (γ_5) symmetry reflects the impossibility of experimental detection of the relative phase of the left and right circularly polarized particles. This impossibility is expressed for the free field through the conservation of the difference between the number of the right and left circularly polarized particles (the law of chirality conservation). For the neutrino field, because of the simple correspondence between Majorana and Weyl descriptions of neutrino, this statement is an equivalent to the condition that comes from the superselection rule for the lepton number.[2] For the electromagnetic field it is an equivalent to the statement about the impossibility of the detection of the absolute plane of polarization of linearly polarized light.

Till now we have not touched upon the question about the role of condition (2.2). But it allows us to give the following physical interpretation of the reason of the existence of the group $U(1)$: chiral (γ_5) symmetry of the theory of massless fields is determined by space-time axial symmetry that is peculiar for the massless particle about an axis that coincides with the direction of its momentum (see, for example, Berestezky et al., (1970)).

Actually, in the group language the space-time axial symmetry is expressed by the fact that for the description of the massless particles with physically meaningful values of helicity we must restrict ourselves to the consideration of the factor group of the "little" group of the Lorentz group[3]:

$$U(R)|\lambda, k\rangle = \exp\{i\Theta[R]J_3\}|\lambda, k\rangle = \exp(i\Theta(R)\lambda)|\lambda, k\rangle \tag{3.3}$$

[2] The neutrino can also be viewed as a purely neutral particle (see, for example, Ryan and Okubo, 1964; Kobzarev and Okun', 1972).

[3] Wigner (see, for example, 1963) defines the "little" group as the subgroup of the Lorentz group consisting of all homogeneous proper Lorentz Transformations $R^\mu{}_\nu$ that do not alter the momentum of the particle: $R^\mu{}_\nu k^\nu = k^\mu$.

where J_3 is a helicity operator, $k_\mu = (0, 0, k, ik)$:

$$J_3 |\lambda, k\rangle = \lambda |\lambda, k\rangle \tag{3.4}$$

and the angle $\Theta[R]$ is some real function of the $R^\mu{}_\nu$.

For infinitesimal $R^\mu{}_\nu$ we have

$$\Theta \to \theta \tag{3.5}$$

and in this case the transformations (3.2) and (3.3) can be identified. If we consider only one particle state the operators Π and J_3 have the same physical meaning.

Unitary extension of the IR of the factor group into the IR of the proper Lorentz group is accompanied by the transition from the states $|\lambda, k\rangle$ to the states $|\lambda, p\rangle$, where $p_\mu = L_\mu{}^\nu p_\nu$. And it follows from the analysis of Weinberg (1964, 1965) that the field operators, which are in mutual correspondence with the states $|\lambda, p\rangle$ and satisfy the equations of motion (2.4) and commutation relations (2.5), are transformed only in accordance with the IR (j_1, j_2) of the proper Lorentz group, which satisfy the condition (2.2).

So, the definition of the γ_5 transformations (2.1) is closely connected with the space-time symmetry of the massless particle and, it is possible to say, it is caused by it. But at the same time it is necessary to warn against mixing transformation (3.2) with (3.3), as sometimes occurs (see, for example, Watanabe, 1957; Perkins, 1972). If the chiral (γ_5) transformations reflect the symmetry of polarization space of a massless particle (it is exactly this symmetry that is meant in the above-mentioned papers), then the transformations (3.3) are induced by the symmetry transformations of the coordinate space.

4. *The Group Structure of the Field Theory of Massless Particles*

In the general case by axial symmetry we understand not only rotations about an axis but also inversions about the planes that are connected with this axis. From this fact it follows that we can discuss the extension of the group of γ_5 transformations $U(1)$. The minimal extension of the abstract group $U(1)$ consists of transition into the group $G = U(1) \otimes_s Z_2$, which is a semisimple product of the group $U(1)$ and the group of its outer automorphisms (see, for example, Lee and Wick, 1966). The group $U(1)$ has only one outer automorphisms: the transition from the element $\exp(i\theta)$ into the element $\exp(-i\theta)$, which is realized by the cyclic group of order 2. For the operator Π the action of the automorphisms is described in the following way:

$$\Pi \to \Pi' = C_\Pi \Pi C_\Pi^{-1} = -\Pi \tag{4.1}$$

From (3.2) one can see that the definition of outer automorphisms (4.1) is connected with the change of helicity of massless particles. We require that field equations (2.4) and commutation relations (2.5) must be invariant under the transformations C_Π of the field operators $\varphi(x)$, $\chi(x)$, which change the helicity of massless particles: $\lambda \to -\lambda$. This condition is satisfied if we identify the operator C_Π with operator CP,[4] where C is charge conjugation, P is space inversion.

As was suggested by Kuo (1971), the outer automorphisms of the internal symmetry groups are themselves symmetries. From this point of view the CP symmetry of the theory of massless particles is a consequence of the conservation of chirality. In the case of massless particles having only integral values of spin, the operator C_Π can be represented by the operator of the space inversion P.

To discuss the relativistic and chiral invariance from the general point of view one must give the transformation properties of the field operators under a 11-parameter group that includes the space-time transformations of the Puankare group $\mathscr{P}$ and γ_5 transformations. In our case, however, it is enough to discuss a 7-parameter group that includes, along with the Lorentz group, the group of γ_5 transformations, and further to enlarge it by the group of space-time displacements. The transformations of this 11-parameter group are described by the usual formulas for the case of space-time transformations [see, for example, Weinberg, 1964, 1965] and formulas (2.1). The generator of chiral transformations Π is a Casimir operator for the IR of the group $\mathscr{P}\otimes U(1)$. The invariants of chiral transformations are transformed according to symmetrical representations of the Lorentz group. For example, the tensor energy–momentum of the electromagnetic field

$$T_{\mu\nu} = -\tfrac{1}{2}(F_{\mu\alpha}F_{\alpha\nu} + \tilde{F}_{\mu\alpha}\tilde{F}_{\alpha\nu}) \tag{3.2}$$

which is transformed in accordance with $\mathrm{IR}(1,1) = (0,1)\otimes(1,0)$, is an invariant of dual transformations of the fields $\mathbf{E}$, $\mathbf{H}$. But the electromagnetic potentials A_μ and Ricci tensor $R_{\mu\nu}$ for the gravitation field are not the invariants of transformations.[5] The reason for that is that IR $(\frac{1}{2}, \frac{1}{2})$ and $(1, 1)$ in these cases do not satisfy the condition (2.2) and because of that are ruled out from the number of IR's of the 7-parameter group. The transformations of the chiral group $U(1)$ are defined only on the physical states of massless particles. But in the case of A_μ and $R_{\mu\nu}$ one must consider unphysical states of massless particles with additional values of helicity.

5. *Neutrino Theory of Light*

From the formal point of view the basic problem of the neutrino theory of light consists in the construction from the neutrino operators of the photon operators that satisfy the usual commutation relations. With some simplification we deal with the construction of electromagnetic potentials A_μ from the neutrino field operators ψ. From the requirement of the right statistical

[4] This can easily be seen by direct examination.

[5] In the paper by Rumer and Fet (1968) the potentials A_μ are treated as dual invariants. This incorrect statement is determined by the absence in their approach of the condition (2.2). The introduction of this condition provides a basis for the physical interpretation of the dual transformations.

properties for the photon the following expression comes out (Barbour et al., 1963):

$$\mathscr{A}_\mu(k) = \int_0^1 f(\lambda)\bar{\psi}((\lambda - 1)k)\gamma_\mu\psi(\lambda, k)d\lambda \tag{5.1}$$

where $\mathscr{A}_\mu(k)$ is the potential in the momentum representation, k_μ is the four momentum of the photon, ψ is an operator of a neutrino field, and $f(\lambda)$ is some weighted function.

Neutrino and photon states realize the IR of chiral group $U(1)$ and all the relations that connect the neutrino and photon operators must have definite transformation properties under chiral transformations. The electromagnetic potentials $\mathscr{A}_\mu(k)$ are not dual invariants. At the same time the right side of the expression (5.1) is an invariant of γ_5 transformations. Of course, from the particular solutions of the Dirac equation with $m = 0$ can be built the transversal four vectors (Berezynsky, 1966). And they will have the correct transformation properties under dual transformations [see (2.11)]; i.e., they are not invariant under transformations of the group $U(1)$. In this case, however, the condition of the relativistic invariance will not be satisfied. So, we can formulate the Pryce theorem in the following way: The requirements of the correct statistics and the correct transformation properties under transformation of the group $U(1)$ of the composite photons are incompatible in the neutrino theory of light. In other words we can say that with the satisfaction of the requirement of the relativistic and chiral invariance one can build from the neutrino only a photon with unphysical longitudinal polarization.

In a paper by Berezynsky (1966) are formulated in a manifest way the principal conditions that are the basis of most papers in the field of the neutrino theory of light. On the basis of our approach these conditions can be given the following interpretation: The operators of the massless fields have the definite transformation rules under chiral (γ_5) transformations; there exist field operators **E**, **H** that are transformed under dual transformations in accordance with (2.11); a neutrino can be either a fermion or a parafermion particle; the commutation relations of the operators of the photon fields are invariants under dual transformations. These conditions correspond, respectively, to conditions (6), (8), (9), (7), (10), and (11) of Berezynsky. The last condition is an equivalent to the condition of the pure neutrality of the photon by Berezynsky. Berezynsky showed that we do not have a self-consistent neutrino theory of light if all these conditions are satisfied. From our point of view, that means that the statistical properties of the photon in the neutrino theory of light are inconsistent with the chiral (γ_5) symmetry of neutrino and electromagnetic fields.

In papers by Perkins (1972), Green (1972), and Inone et al. (1973) that appeared after the paper by Berezynsky (1966) was published, there was made an attempt, by some modifications of the commutation relations for the electromagnetic field, to get round the difficulties that were mentioned above. It is not the aim of the present work to give the detailed analysis of the results of these papers. But we can note that these modifications of the theory lead to the dual noninvariant formulations, and from the point of view developed above this fact is not satisfactory.

6. *Conclusion*

The interest in the internal symmetries of the theory of massless fields is caused by the fact that they defined to such a great extent the formulations of the theories that describe the interactions with massless particles. It is enough to remember about the role of γ_5 symmetry in the construction of the theory of weak interactions. The consideration of the dual symmetry in the presence of charged particles also leads to interesting results (see, for example, Strazhev and Tomilchik 1973, 1975). The question about the local chiral (γ_5) transformations should be paid attention to. In papers by Misner and Wheeler (1957) and Collinson and Shaw (1972) the localization of parameter of chiral transformations was used for the geometrization of electromagnetic and neutrino fields.

This approach was also used for the discussion of γ_5 symmetry of spinor field that describes particles with mass (Hosek, 1972). But the investigations in this direction are only a beginning.

The consideration of the γ_5 symmetry of massless fields from the general point of view may be very interesting in connection with the development of the unified theory of weak and electromagnetic interactions (see, for example, Bernstein, 1974). But further understanding of this question requires the development of the theory of gauge fields in the presence of two types of sources (see, for example, Hooft, 1974; Klimo and Dowker, 1973).

Note Added in Manuscript. As I have learned, in the papers of Deser and Teitelboim (1976) [*Physical Review D*, **13**, 1592] and Weaver (1976) [*Annals of Physics*, **101**, 52] some results of Section 2 of the present paper are repeated.

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